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Optimizing Support Structures for Tensile Architecture

David Jourdan[1], Mélina Skouras[2], Adrien Bousseau[1]

[1]Inria, Université Côte d'Azur
[2]Inria, Univ. Grenoble Alpes, CNRS, Grenoble INP, LJK, France

Résumé
Tensile structures are architectural shapes made of stretched elastic material that can be used to create large-span roofs. Their elastic properties make it quite challenging to obtain a specific shape, and the final shape of a tensile structure is usually found rather than imposed.
We present a design tool for tensile structures that, unlike existing software, lets the user specify the shape they want and finds the closest fit. From an input height field, the method finds a tensile structure that best approximates the target. Based on a mass-spring formulation, we use non-linear optimization tools to find a sparse set of forces which, when applied to an elastic membrane, creates a tensile surface as close as possible to the input.

Mots clé : Tensile structures, architecture, computational design, inverse design

1. Introduction

Tensile structures and their intriguing shapes have sparked the interest of architects and engineers alike for decades (see, e.g. figure 1). Since these structures are only composed of a rigid support and a lightweight, elastic membrane, they are well-suited to create large-span roofs. However, architects and engineers cannot directly control their shape the way they would do with rigid structures such as masonry, because the shape of the membrane naturally arises from its internal elasticity, so it can only be controlled by moving the support structure (i.e. the cables or the masts). The final shape of a tensile structure can be visualized by physically prototyping it, or by running a simulation, a process that is called form-finding (see e.g. the report on form-finding algorithms for tension structures by Veenendaal & Block [VB12]).

Because of this form-finding process, designing a tensile structure is a tedious, trial-and-error process which usually involves running simulations for many different parameters until the desired shape is reached. Our goal is thus to develop an easier and more intuitive tool to design tensile structures. Instead of setting up a set of specific parameters such as the position of masts or the internal elasticity of the membrane and computing the resulting shape, we do the opposite and find the support structure that makes it possible to reach the desired shape.

Such an approach can be labeled as *inverse design*, as opposed to forward modeling, or forward design. Usually the inverse problem is much more difficult to solve than the direct one, there can be an infinite number of possible configurations leading to the same solution, hence the need for regularization.

FIGURE 1 – *The Dance Pavilion*, Frei Otto

Our algorithm takes as input a given target shape and returns a tensile surface that best approximates it, as well as a support structure composed of masts. We add a sparse regularizer in our formulation to guide the optimization towards solutions with fewer masts, which makes for a lighter support structure. Note that the result of our method may not be exactly the same as the target shape because tensile surfaces belong to a very specific class of shapes so if the target surface is not itself equivalent to a tensile surface then the result of the optimization will be different from the input.

2. Related work

2.1. Form-finding of Tensile structures

A lot of work has been done on forward simulation of tensile structures, and we can classify these form-finding algorithms in 3 categories, as pointed out in the Veenendaal & Block report [VB12] as well as in Tension Structures [Lew03].

The first class is based on the *force-density method* [LS72] which was first developed for cable nets, and later adapted to elastic membranes, Bletzinger & Ramm [BR99] added triangular finite elements to the formulation so that the method generalizes to a continuous elastic membrane. These methods differ by the solving strategies used but both are proven to be equivalent to the simpler, cable net case. In the multi-step force-density method [SSM07] the simpler model of a cable net and its associated linear formulation is used for faster prototyping.

The second family of methods is based on the *dynamic relaxation method* [Bar88]. This method considers the dynamic problem, it consists in solving Newton's Second Law of Motion for every vertex of the mesh such that its speed has to be equal to 0 since we want the shape at static equilibrium. This is an iterative method that solves for successive residuals and adds a damping coefficient for accelerated convergence.

The third class is based on the *stiffness matrix method* which is a standard method in structural analysis for computing small displacements in the structure with respect to certain loads. It has then been modified to compute larger displacements and applied to the form-finding of tensile structures [AAB74]. We use a Newton-Raphson scheme with a mass-spring energy to simulate a tensile structure in our application. Even if our method solves the inverse problem instead of the direct one, our model belongs to this class of iterative methods.

2.2. Computational design

The area of computational design received a lot of attention in the graphics community recently. Its goal is to develop tools to help the designer, artist, or architect to focus on the design and to let the computer take care of more practical considerations such as structural stability, materials or feasibility. Such tools can help to shorten the design loop where a project goes back and forth between the creative team which focuses on the design, and the technical team which analyses it.

Deformable materials In the context of fabrication, several methods have been developed for approximating a given 3D shape with different materials and fabrication techniques. In particular, creating shapes from flat, elastic pieces of material is of interest because it greatly reduces fabrication costs. However, the use of elastic and deformable material requires to take into account complex, non-linear behavior. Elastic or deformable material thus requires an *inverse problem* to find a shape that matches a target when certain forces act on it.

Skouras et al. optimized the shape of a rubber balloon so that it matches a given shape when inflated [STBG12], likewise, they optimized the shape of flat panels so that when seamed together they form an inflatable structure which matches a given shape when inflated [STK*14]. Instead of using air pressure to deform a given object, one can directly use the internal forces of a pre-stretched elastic membrane. In Guseinov et al. [GMB17] rigid tiles are enclosed between two elastic sheets so that when the tiles are pulled together by the elastic forces, the assembly can go out of plane. The simulation of elastic membranes has been investigated in more detail by Perez et al. [POT17], even though the inverse problem has been tackled only for local editing. Moreover, the problem of automatically determining the support structure, which correspond to discrete degrees of freedom in the optimization, has not been tackled.

Architecture Computational design of architectural structures has received a lot of interest in computer graphics. Geometric approaches have been developed in a recent field called architectural geometry, while methods taking into account static mechanics and structural stability have been developed for self-supporting structures.

To have a complete overview of the field of architectural geometry, we refer to the report by Pottmann et al. [PEVW15]. In the context of fabrication-aware design, architectural geometry is interested in approximating freeform surfaces by meshes optimized for fabrication, which is often called rationalization in architecture. Rationalization consists in approximating a freeform architectural surface with elements that are easy or cheap to fabricate. For example in Eigensatz et al. [EKS*10] a freeform surface is approximated by a union of patches, called panels, while minimizing fabrication costs. The idea is that flat panels are the cheapest to produce since they can all be cast on the same mold, but they do not approximate well curved surfaces, so some curved ones are needed, such as cylindric panels which are developable, and paraboloids, torus patches and cubic patches that are doubly-curved.

Aside from these purely geometric methods, some work has been done on integrating static mechanics and structural analysis on the design of architectural shapes. For self-supporting masonry, the Thrust Network Analysis framework [BO07] enabled the development of tools aimed at designing freeform self-supporting structures. The topic has been introduced to graphics by Vouga et al. [VHWP12] who linked it to discrete differential geometry, and proposed design and editing tools for self-supporting surfaces, and further developed by Panozzo et al. [PBSH13] who solved the problem of finding the closest self-supporting structure to a given target surface. Some specific problems related to these structures have also been studied, such as optimizing the amount of material used [KPWP17], or their assembly [DPW*14], but to our knowledge, the design of tensile structures has not been studied in such a comprehensive way, and the problem of finding the closest tensile structure to an input surface has yet to be solved.

3. Method

3.1. Mathematical formulation

Given a target mesh $\mathcal{M}$ we want to find the vertex positions $\mathbf{x} = (x_1^\top, \dots, x_n^\top)^\top$ of the membrane, and external forces $\mathbf{p} = (p_1^\top, \dots, p_n^\top)^\top$ corresponding to masts, which minimize the distance between $\mathbf{x}$ and $\mathcal{M}$. This is a constraint optimization problem, the unknowns $\mathbf{x}$ and $\mathbf{p}$ need to respect the *equilibrium condition*, that is the sum of the internal membrane forces $\mathbf{f}(\mathbf{x})$ and the external forces $\mathbf{p}$ must be equal to zero. The internal membrane forces are computed as mass-spring forces, with mass being lumped at vertex positions and springs corresponding to edges of the membrane mesh.

Such a problem is unfortunately ill-posed. Even with the equilibrium condition satisfied there can be an infinite number of solutions, forces in $\mathbf{p}$ can have arbitrarily high magnitudes that compensate each other while preserving the shape of the input. We thus need to guide the optimization with a regularization term $R_{sparse}(\mathbf{p})$, which will favor solutions where $\mathbf{p}$ is sparse.

This is formulated as :

$$\begin{aligned} &\min_{\mathbf{x},\mathbf{p}} E_{\text{dist}}(\mathbf{x}) + R_{sparse}(\mathbf{p}) \\ &\text{s.t. } \mathbf{f}(\mathbf{x}) + \mathbf{p} = 0 \end{aligned} \tag{1}$$

where E_{dist} is the sum of pairwise distances between vertices of the target mesh $\hat{\mathbf{x}}$ and vertices of the membrane mesh $\mathbf{x}$

$$E_{\text{dist}}(\mathbf{x}) = \sum_i ||x_i - \hat{x}_i||^2 \tag{2}$$

and $\mathbf{f}(\mathbf{x})$ are the internal forces of the membrane computed as mass-spring forces.

Since each external force corresponds to a mast, and we want light tensile structures with a few number of masts, the preferred solutions should have a small number of non-zero forces. We thus choose a regularizer that penalizes the sum of the magnitude of the forces, as was previously done by Skouras et al. [STC*13] :

$$R_{sparse}(\mathbf{p}) = k_{sparse} \sum_i ||p_i||_2^{\alpha} \tag{3}$$

with k_{sparse} a scaling parameter and $\alpha \leq 1$ an exponent that generalizes the mixed ℓ_2/ℓ_1 norm to more strongly penalize small values.

To make this regularizer differentiable at 0, we need to add a small constant ε to it :

$$R_{sparse}(\mathbf{p}) = k_{sparse} \sum_i (||p_i||^2 + \varepsilon)^{\frac{\alpha}{2}}$$

3.2. Optimization

The constrained problem (équation 1) is turned into an unconstrained one using the Augmented Lagrangian Method, which is then itself solved using the Newton-Raphson scheme. For details we refer to Nocedal & Wright [NW06].

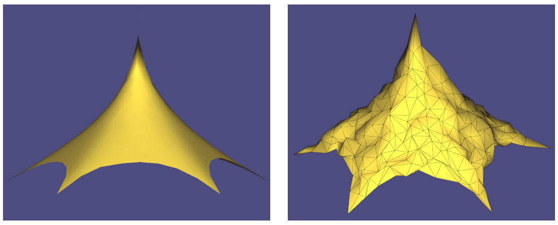
However, after solving équation 1, a few close-to-zero forces may still be present. Since these forces are perturbing the result, we run a second round of optimization after removing variables corresponding to forces close to 0, in the spirit of Iterative Rounding methods (see figure 3).

4. Results

We first tested our method on a tent model, obtained through forward simulation, that we knew could be reached exactly with our method. We also tested if we could recover the same model with added noise (figure 2).

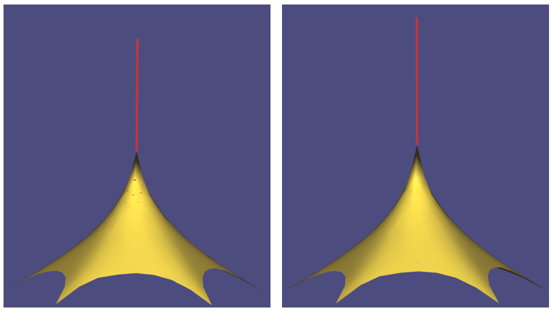
The results in figure 3 show that the iterative rounding step was necessary even for the simplest example where the target surface is a membrane obtained through forward simulation, even if the method appropriately locates the position of the external force, its magnitude is incorrect, because neighboring vertices are influenced by forces of lesser magnitude. In figure 4, the iterative rounding step also helps finding a more realistic and "tensile-like" result.

We then tested our method on a surface that cannot be fabricated with elastic membranes, a model with 3 convex bumps. As shown in figure 5, the bumps are difficult to reproduce with a sparse set of forces, thus the algorithm has to find a compromise between minimizing the distance between the target and the membrane on one hand, and imposing sparsity on the other hand. Since the boundary of the target surface is flat, we had to remove the variables corresponding to the boundary from the optimization, because it introduces too much noise in the result. After a few rounds of iterative rounding, the algorithm manages to find a solution with a sparse set of forces.



(a) (b)

FIGURE 2 – Test model with and without noise



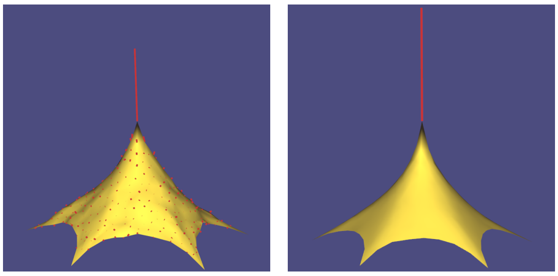
(a) (b)

FIGURE 3 – *Before and after iterative rounding*, in (a) a few close-to-zero forces are perturbing the result, in (b) we get a more precise value of the mast force

5. Conclusion and Future Work

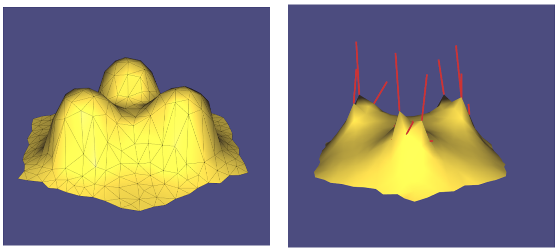
We developed a novel computational design method, using inverse simulation of elastic membranes. Based on a constrained, non-linear optimization problem we used sparsity-inducing regularizers to drive the optimization to solutions which can be fabricated as tensile structures, with some elastic fabric and a few masts.

This method could be extended to integrate other types of support structures such as arches or cables. To make these



(a) (b)

Figure 4 – Noisy model, before and after iterative rounding



(a) Target model (b) Result

Figure 5 – Result on a model with convex bumps

supports emerge, we would need another type of regularizer that would encourage the alignment of forces along curves. We could guide the optimization by analyzing the curvature of the target surface to predict where to place such arches or cables.

Références

[AAB74] Argyris J., Angelopoulos T., Bichat B. : A general method for the shape finding of lightweight tension structures. *Computer Methods in Applied Mechanics and Engineering. Vol. 3*, Num. 1 (1974), 135 – 149.

[Bar88] Barnes M. R. : Form-finding and analysis of prestressed nets and membranes. *Computers & Structures. Vol. 30*, Num. 3 (1988), 685–695.

[BO07] Block P., Ochsendorf J. : Thrust network analysis : A new methodology for three-dimensional equilibrium. *Journal of the International Association for shell and spatial structures. Vol. 48*, Num. 3 (2007), 167–173.

[BR99] Bletzinger K.-U., Ramm E. : A general finite element approach to the form finding of tensile structures by the updated reference strategy. *International Journal of Space Structures. Vol. 14*, Num. 2 (1999), 131–145.

[DPW*14] Deuss M., Panozzo D., Whiting E., Liu Y., Block P., Sorkine-Hornung O., Pauly M. : Assembling self-supporting structures. *ACM Trans. Graph.. Vol. 33*, Num. 6 (novembre 2014), 214 :1–214 :10.

[EKS*10] Eigensatz M., Kilian M., Schiftner A., Mitra N. J., Pottmann H., Pauly M. : Paneling architectural freeform surfaces. *ACM transactions on graphics (TOG). Vol. 29*, Num. 4 (2010), 45.

[GMB17] Guseinov R., Miguel E., Bickel B. : Curveups : Shaping objects from flat plates with tension-actuated curvature. *ACM Transactions on Graphics (TOG). Vol. 36*, Num. 4 (2017), 64.

[KPWP17] Kilian M., Pellis D., Wallner J., Pottmann H. : Material-minimizing forms and structures. *ACM Transactions on Graphics (TOG). Vol. 36*, Num. 6 (2017), 173.

[Lew03] Lewis W. : *Tension Structures Form and Behaviour*. Thomas Telford, London, 2003.

[LS72] Linkwitz K., Schek H. : A new method of analysis of prestressed cable networks and its use to the roofs of the olympic games facilities in munich. *9th Congress of IABSE* (1972).

[NW06] Nocedal J., Wright S. J. : *Numerical Optimization, 2nd ed.* Springer-Verlag, New York, 2006.

[PBSH13] Panozzo D., Block P., Sorkine-Hornung O. : Designing unreinforced masonry models. *ACM Trans. Graph.. Vol. 32*, Num. 4 (juillet 2013), 91 :1–91 :12.

[PEVW15] Pottmann H., Eigensatz M., Vaxman A., Wallner J. : Architectural geometry. *Computers & graphics. Vol. 47* (2015), 145–164.

[POT17] Pérez J., Otaduy M. A., Thomaszewski B. : Computational design and automated fabrication of kirchhoff-plateau surfaces. *ACM Trans. Graph.. Vol. 36*, Num. 4 (juillet 2017), 62 :1–62 :12.

[SSM07] Sánchez J., Serna M. Á., Morer P. : A multi-step force-density method and surface-fitting approach for the preliminary shape design of tensile structures. *Engineering Structures. Vol. 29*, Num. 8 (2007), 1966–1976.

[STBG12] Skouras M., Thomaszewski B., Bickel B., Gross M. : Computational design of rubber balloons. In *Computer Graphics Forum* (2012), vol. 31, Wiley Online Library, pp. 835–844.

[STC*13] Skouras M., Thomaszewski B., Coros S., Bickel B., Gross M. : Computational design of actuated deformable characters. *ACM Trans. Graph.. Vol. 32*, Num. 4 (juillet 2013), 82 :1–82 :10.

[STK*14] Skouras M., Thomaszewski B., Kaufmann P., Garg A., Bickel B., Grinspun E., Gross M. : Designing inflatable structures. *ACM Trans. Graph.. Vol. 33*, Num. 4 (juillet 2014), 63 :1–63 :10.

[VB12] Veenendaal D., Block P. : An overview and comparison of structural form finding methods for general networks. *International Journal of Solids and Structures. Vol. 49*, Num. 26 (2012), 3741–3753.

[VHWP12] Vouga E., Höbinger M., Wallner J., Pottmann H. : Design of self-supporting surfaces. *ACM Transactions on Graphics (TOG). Vol. 31*, Num. 4 (2012), 87.